

## ☺ 1.5 – Loans and Investments ☺

### Daily Objectives

1. Use recursive models to understand and model financial situations
2. Explore compound interest scenarios
3. Use recursive routines to model loans, credit card scenarios, and investments

**Example 1:** Jimmy deposits \$400 into a savings account. How much money will he have after 3 years?

↳ earning 4.4% annual interest

$$400 \times 1.044 \times 1.044 \times 1.044 = \boxed{\$455.16}$$

Some banks compound interest quarterly or monthly. When this happens, we divide the interest rate by the number of times the interest is compounded each year.

**Example 2:** Gloria deposits \$2,000 into a bank that pays 7% interest compounded *quarterly*.

- a. Write a recursive routine to represent the amount of money Gloria will have after  $n$  quarter-years.

$$u_0 = 2000$$

$$u_n = \left(1 + \frac{.07}{4}\right) u_{n-1}$$

$$n \geq 1$$

- b. How much money will Gloria have after 1 year?

$$2000 \times 1.0175 \times 1.0175 \times 1.0175 \times 1.0175 = \boxed{\$2143.72}$$

- c. Write a recursive routine to represent Gloria's money if her interest is compounded *monthly*. How much money will Gloria have after 1 year?

$$u_0 = 2000$$

$$u_n = \left(1 + \frac{.07}{12}\right) u_{n-1}$$

$$n = \# \text{ of months } n \geq 1$$

$$2000 (1.00583)^{12} = \boxed{\$2144.58}$$

- d. Write a recursive routine to represent Gloria's money if her interest is compounded *weekly*? How much money will Gloria have after 1 year?

$$u_0 = 2000$$

$$u_n = \left(1 + \frac{.07}{52}\right) u_{n-1}$$

$$n = \# \text{ of weeks } n \geq 1$$

$$2000 (1.0013461538)^{52}$$

$$\boxed{\$2144.92}$$

**Example 3:** You take out a loan for \$10,000 to buy a car! The bank gives you an interest rate of 4.5% compounded monthly. You decide to pay \$200 a month

- a. Write a recursive routine for this situation:

$$u_0 = 10,000$$

$$u_n = \left(1 + \frac{0.045}{12}\right) u_{n-1} - 200$$

$$n \geq 1$$

- b. How much will you still owe after 1 year?

$$\{0, 10,000\}$$

$$\{Ans(1)+1, Ans(2) \times \left(1 + \frac{0.045}{12}\right) - 200\}$$

(12, \$8009.27)

\$ 8009.27

- c. How long will it take you to pay off your loan?

(56, -105.06)

56 months

- d. What would you have to pay per month if you wanted to pay your loan off in four years?

\$ 228.04

- e. [Continuation of part d] How much did you end up paying for your car?

$$47 \times 228.04 + 227.77$$

$$47,226.92$$

$$48, -269.77$$

\$ 10,945.65

overpayment

$$228.04 - .27 = 227.77$$

What recursive formula would you write for a loan or investment with principal \$1,000 and annual interest rate of 12%, compounded monthly, with monthly payments of \$50?

$$u_0 = 1,000$$

$$u_n = \left(1 + \frac{0.12}{12}\right) u_{n-1} - 50$$

$n = \text{number of months}$

$$n \geq 1$$